Indian J. Phys. **47**, 641–649 (1973)

Impossibility of reduction of generalized electromagnetic fields for non-zero mass system in free space

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(*Received* 18 *July* 1972, *revised* 24 *October* 1972)

Symmetrical reduced expansions of electric and magnetic fields are derived in the presence of electric charges and Dirac's monopoles in terms of irreducible representations of proper, orthochronous, inhomogeneous Loretz group for non-zero mass system. Maxwell's field equations are modified by considering the generalized charge as a complex quantity with electric and magnetic fundamental charges as its real and imaginary parts. The constancy condition for the ratio of electric and magnetic charges on a particle has been imposed for explaining the results obtained by the requirement that generalized electromagnetic fields satisfy generalized field equations. Considering the reduced expansions in the absence of electric or magnetic charge source, it has been proved that the derivation of reduced expansions for electromagnetic fields satisfying the field equatons without sources is not possible for non-zero mass system.

Introduction

The recipe given by Lomont & Moses (1967) enables one to reduce any unitary ray representation of proper, orthochronous, inhomogeneous Lorentz group for non-zero (Moses 1967) and zero (Moses 1968) mass systems in terms of Foldy Shirokov (1956, 1958) and Lomont-Moses (1964) realizations respectively Using these techniques, we could derive the reductions of wavefunctions which transform as scalar field for zero-mass system (Rajput 1969a) and antisymmetric complex tensor (Rajput 1969b, 1969c) and three-vector (Rajput 1969d) fields for non-zero and zero-mass systems. These reductions have been applied to derive the reduced expansions of electromagnetic fields in linear (Rajput 1970a)and angular momentum basis (Rajput 1969e). For non-zero mass system, these reduced expansions of electric and magnetic fields have been derived in the following forms (Rajput 1969c).

$$\boldsymbol{E} = \frac{i}{4\pi^{3/2}} \sum_{\epsilon=\pm 1} \epsilon \int d\boldsymbol{p} \left[mf(\epsilon, \boldsymbol{p}) + \frac{\boldsymbol{p}\{\boldsymbol{p}.f(\epsilon, \boldsymbol{p})\}}{w(p)+m} \right] \times \exp[i\{\boldsymbol{p}.\boldsymbol{x} - \epsilon w(p)t\}] \quad \ldots (1)$$

$$\boldsymbol{H} = \frac{i}{4\pi^{3/2}} \sum_{\epsilon=\pm 1} m \int \frac{d\boldsymbol{p}}{w(p)} \boldsymbol{p} \times f(\epsilon, \boldsymbol{p}) \exp[i\{\boldsymbol{p}.\boldsymbol{x} - \epsilon w(p)t\}], \quad \ldots (2)$$

where the vector functions $f(\epsilon, \boldsymbol{p})$ are the representations of wave-function which transforms as electromagnetic fields, in the basis characterized by the space of wave-function in Hilbert space, upon which the generators of proper, orthochronous, inhomogeneous Lorentz group operate; m is the eigenvalue of mass operator; ϵ is the sign of energy ($\epsilon = \pm 1$), and $w(p)$ is given by

$$w(p) = (m^2+p^2)^{\frac{1}{2}}, |\boldsymbol{p}| = p$$

These reduced expansions of electric and magnetic fields are not symmetrical at all, since the expansion of $\boldsymbol{E}$ consists of two parts one of which is parallel to the momentum vector $\boldsymbol{p}$ and the other to the vector wavefunction $f(\epsilon, \boldsymbol{p})$ while that of $\boldsymbol{H}$ is purely transverse having only one term whch is perpendicular to both $\boldsymbol{p}$ and $f(\epsilon, \boldsymbol{p})$ This lack of symmetry between electric and magnetic fields in the presence of electric charge source can be removed if these reduced expansions are derived in the presence of electric charge and Dirac's magnetic monopole (1931, 1948).

Considering the generalized fundamental charge as complex quantity with electric and magnetic fundamental charges as its real and imaginary parts, the reduction of generalized electromagnetic fields has already been derived for zero-mass system (Rajput 1970b) Furthermore it has been shown (Prakash & Rajput, communicated) that the values of coupling parameters are not altered by considering generalized charge as a complex quantity in this manner.

In the present work we undertake the study of reduction of generalized electromagnetic fields to the irreducible representation of orthochronous, proper, inhomogeneous Lorentz group. Symmetrical reduced expansions are derived for generalized electric and magnetic fields in the presence of electric and magnetic charges It is proved that the longitudinal component of electric or magnetic field is zero in the absence of corresponding charge source. Conditions, that the reduced expansion of wave function, which transforms as generalized electromagnetic field satisfies generalized Maxwell's field equations, have been derived and it is found that the electric and magnetic charge densities are proportional to the corresponding current densities. For simplicity, the ratio of fundamental electric and magnetic charges is taken as a constant

Considering the reduced expansions in absence of electric or magnetic charge source, it has been proved that the derivation of reduced expansions for electromagnetic fields satisfying the field equations without sources, is not possible for non-zero mass system.

Reduction of Generalized Electromagnetic Fields

Wavefunction ψ which transforms as electromagnetic fields is given by

$$\psi(\boldsymbol{x}, t) = \boldsymbol{E} - i\boldsymbol{H} \quad \dots \quad (3)$$

Correspondingly, the generalized charge q in the presence of electric charge e and magnetic charge g, may be considered as

$$q = e - ig$$
$$|q| = (e^2+g^2)^{\frac{1}{2}} \qquad \ldots (4)$$

Epstein (1967) derived the values of electric and magnetic fields in the presence of electric and magnetic charges, in terms of vector potential $\boldsymbol{A}$ and scalar potential ϕ. These results, for $c = \hbar = 1$, can be written as

$$\boldsymbol{E} = -\frac{e}{|q|}(\dot{\boldsymbol{A}}+\nabla\phi) - \frac{g}{|q|}\nabla\times\boldsymbol{A}, \qquad \ldots (5)$$

$$\boldsymbol{H} = \frac{e}{|q|}(\nabla\times\boldsymbol{A}) - \frac{g}{|q|}(\dot{\boldsymbol{A}}+\nabla\phi). \qquad \ldots (6)$$

For the reductions of these fields to the irreducible representations of inhomogeneous, proper, orthochronous Lorentz group, we use the following reduced expansions of scalar (1967) and vector (1969c) fields for non-zero mass system :

$$\phi = \sum_{\epsilon=\pm1} \int c(\epsilon) \frac{d\boldsymbol{p}}{w(p)} f^0(\epsilon, \boldsymbol{p}) \exp[i\{\boldsymbol{p}\,\boldsymbol{x} - \epsilon w(p)t\}] \qquad \ldots (7)$$

$$\boldsymbol{A} = \sum_{\epsilon=\pm1} c(\epsilon) \int \frac{d\boldsymbol{p}}{w(p)} \exp[i\{\boldsymbol{p}.\boldsymbol{x} - \epsilon w(p)t\}]$$
$$\times\left[mf^1(\epsilon, \boldsymbol{p}) + \frac{\boldsymbol{p}\{\boldsymbol{p}\,f^1(\epsilon, \boldsymbol{p})\}}{w(p)+m}\right] \qquad \ldots (8)$$

where $f^0(\epsilon, \boldsymbol{p}) = f^0(m, \epsilon, \boldsymbol{p})$ and $f^1(\epsilon, \boldsymbol{p}) = f^1(m, \epsilon, \boldsymbol{p})$ are the representations of wavefunctions of particles with spin zero and one respectively, in the basis characterized by the Hilbert space on which the generators of inhomogeneous Lorentz group operate Substituting these values of $\boldsymbol{A}$ and ϕ in eqns. (5) and (6), the following reductions of generalized electric and magnetic fields to the irreducible representations of inhomogeneous Lorentz group may be derived in the presence of both electric and magnetic charges :

$$\boldsymbol{E} = -\frac{ie}{|q|}\sum_{\epsilon=\pm1} c(\epsilon) \int \frac{d\boldsymbol{p}}{w(p)} \boldsymbol{p} f^0(\epsilon, \boldsymbol{p}) \exp[i\{\boldsymbol{p}\,\boldsymbol{x} - \epsilon w(p)t\}]$$
$$+\frac{i}{|q|}\sum_{\epsilon=\pm1} c(\epsilon) \int d\boldsymbol{p}\, \exp[i\{\boldsymbol{p}.\boldsymbol{x} - \epsilon w(p)t\}][\epsilon e m f^1(\epsilon, p)$$
$$+\frac{\epsilon e \boldsymbol{p}\{\boldsymbol{p}.f^1(\epsilon, \boldsymbol{p})\}}{w(p)+m} - \frac{gm}{w(p)}\boldsymbol{p}\times f^1(\epsilon, \boldsymbol{p})] \qquad (9)$$

and

$$\boldsymbol{H} = -\frac{ig}{|g|}\sum_{\varepsilon=\pm 1} c(\epsilon)\int \frac{d\boldsymbol{p}}{w(p)}\,\boldsymbol{p} f^0(\epsilon, \boldsymbol{p})\exp[i\{\boldsymbol{p}.\boldsymbol{x}-\epsilon w(p)t\}]$$

$$+\frac{i}{|g|}\sum_{\varepsilon=\pm 1} c(\epsilon)\int d\boldsymbol{p}\exp[i\{\boldsymbol{p}.\boldsymbol{x}-\epsilon w(p)t\}]$$

$$\times\left[g\epsilon m f^1(\epsilon, \boldsymbol{p})+\frac{g\epsilon\boldsymbol{p}\{\boldsymbol{p}.f^1(\epsilon, \boldsymbol{p})\}}{w(p)+m}+\frac{\epsilon m}{w(p)}\{\boldsymbol{p}\times f^1(\epsilon, \boldsymbol{p})\}\right] \quad \ldots \quad (10)$$

Using these reduced expansions in eqn. (3), we get the reduction of wavefunction which transforms as the generalized electromagnetic fields.

$$\psi(x) = -\frac{iq}{|q|}\sum_{\varepsilon=\pm 1} c(c)\int d\boldsymbol{p}\,\boldsymbol{p} f^0(\epsilon, p)\exp[i\{\boldsymbol{p}.\boldsymbol{x}-\epsilon w(p)t\}]$$

$$+\frac{iq}{|q|}\sum_{\varepsilon=\pm 1} c(\epsilon)\int d\boldsymbol{p}\exp[i\{\boldsymbol{p}.\boldsymbol{x}-\epsilon w(p)t)\}]$$

$$\times\left[\epsilon m f^1(\epsilon, \boldsymbol{p})+\frac{\boldsymbol{p}\{\boldsymbol{p}.f^1(\epsilon\ \boldsymbol{p})\}}{w(p)+m}-\frac{im}{w(p)}\boldsymbol{p}\times f^1(\epsilon, \boldsymbol{p})\right] \quad \ldots \quad (11)$$

Maxwell's field equations are modified in the following generalized form in the presence of electric and magnetic charge and current densities :

$$\nabla\cdot\boldsymbol{\Psi} = 4\pi J_0 \quad \ldots \quad (12)$$

$$\nabla\times\boldsymbol{\Psi} = -\frac{i\partial\psi}{\partial t}-4\pi i\boldsymbol{J}, \quad \ldots \quad (13)$$

where J_0 and $\boldsymbol{J}$ are generalized charge and current densities defined in terms of generalized current four-vector J_α which may be written as a complex quantity with electric and magnetic current four-Vectors, j_α and k_α, as its real and imaginary parts,

$$J_\alpha(x) = j_\alpha(x)-ik_\alpha(x), \quad \ldots \quad (14)$$

where

$$j_\alpha(x) = \begin{pmatrix} j_0(x) \\ \boldsymbol{j}(x) \end{pmatrix};$$

$$k_\alpha(x) = \begin{pmatrix} k_0(x) \\ \boldsymbol{k}(x) \end{pmatrix}.$$

The wavefunction $\psi(x)$ given by equation (11) may be considered as a sum of longitudinal and transverse parts, ψ^L and ψ^T, which are given by the following reduced expansions :

$$\psi^L = -\frac{iq}{|q|}\sum_{\epsilon=\pm 1} c(\epsilon)\int \frac{d\boldsymbol{p}}{w(p)}\,\boldsymbol{p}f^0(\epsilon,\boldsymbol{p})\exp\,[i\{\boldsymbol{p}.\boldsymbol{x}-\epsilon w(p)t\}]$$

$$+\frac{iq}{|q|}\sum_{\epsilon=\pm 1} c(\epsilon)\int d\boldsymbol{p}\;\exp\,[i\{\boldsymbol{p}\;\boldsymbol{x}-\epsilon w(p)t\}]$$

$$\times\left[\,mf^1(\epsilon,\boldsymbol{p})+\frac{\boldsymbol{p}\{\boldsymbol{p}.f^1(\epsilon,\boldsymbol{p})\}}{w(p)+m}\,\right] \qquad \ldots \quad (15)$$

and

$$\psi^T = \frac{mq}{|q|}\sum_{\epsilon=\pm 1} c(\epsilon)\int \frac{d\boldsymbol{p}}{w(p)}\{\boldsymbol{p}\times f^1(\epsilon,\boldsymbol{p})\{\exp\,[i\{\boldsymbol{p},\boldsymbol{x}-\epsilon w(p)t\}]$$

$$= \text{curl } V^T, \qquad \ldots \quad (16)$$

where

$$V^T = -\frac{imq}{|q|}\sum_{\epsilon=\pm 1} c(\epsilon)\int \frac{d\boldsymbol{p}}{w(p)} f^1(\epsilon,\boldsymbol{p})\exp[i\{\boldsymbol{p}\;\boldsymbol{x}-\epsilon w(p)t\}]$$

Using equation (3) and (16), the transverse parts of generalized electric and magnetic fields can be written as

$$E^T = -\nabla\times A^T,\; H^T = \nabla\times B^T \qquad \ldots \quad (17)$$

where

$$A^T = -\frac{mq}{|q|}\sum_{\epsilon=\pm 1} c(\epsilon)\int \frac{d\boldsymbol{p}}{w(p)} f^1(\epsilon,\boldsymbol{p})\exp\,[i\{\boldsymbol{p}.\boldsymbol{x}-\epsilon w(p)t\}] \qquad \ldots \quad (18)$$

$$B^T = \frac{me}{|q|}\sum_{\epsilon=\pm 1} c(\epsilon)\int \frac{d\boldsymbol{p}}{w(p)} f^1(\epsilon,\boldsymbol{p})\exp\,[i\{\boldsymbol{p}.\boldsymbol{x}-\epsilon w(p)t\}] \qquad \ldots \quad (19)$$

It follows from these equations that the reduced expansions for the vector potentials of transverse electric and magnetic fields are similar except that q in A^T is replaced by $-e$ in B^T, while Cabbibo & Ferrari (1962) assumed them as two different potentials. The longitudinal parts of generalized electromagnetic fields for non-zero mass system, are then readily derived from eqn (15) in the following forms :

$$E^L = -\frac{ie}{|q|}\sum_{\epsilon=\pm 1} c(\epsilon)\int \frac{d\boldsymbol{p}}{w(p)}\,\boldsymbol{p}f^0(\epsilon,\boldsymbol{p})\exp\,[i\{\boldsymbol{p}.\boldsymbol{x}-\epsilon w(p)t\}]$$

$$+\frac{ie}{|q|}\sum_{\epsilon=\pm 1} c(\epsilon)\int d\boldsymbol{p}\exp\,[i\{\boldsymbol{p}.\boldsymbol{x}-\epsilon w(p)t\}]$$

$$\times\left[\quad {}^1(\epsilon,\boldsymbol{p})+\frac{\boldsymbol{p}\{\boldsymbol{p}.f^1(\epsilon.\boldsymbol{p})\}}{w(p)+m}\right] \qquad \ldots \quad (20)$$

$$H^L = -\frac{ig}{|q|} \sum_{\epsilon=\pm 1} c(\epsilon) \int \frac{d\boldsymbol{p}}{w(p)} \boldsymbol{p} f^0(\epsilon, \boldsymbol{p}) \exp[i\{\boldsymbol{p}.\boldsymbol{x} - \epsilon w(p)t\}]$$

$$+ \frac{ig}{|q|} \sum_{\epsilon=\pm 1} c(\epsilon) \int d\boldsymbol{p} \exp[i\{\boldsymbol{p}.\boldsymbol{x} - \epsilon w(p)t\}]$$

$$\times \left[m f^1(\epsilon, \boldsymbol{p}) + \frac{\boldsymbol{p}\{\boldsymbol{p}\, f^1(\epsilon, \boldsymbol{p})\}}{w(p) + m} \right] \quad \ldots \quad (21)$$

It is clear from these equations that the longitudinal parts of electric and magnetic fields are proportional to e and g and therefore the longitudinal component of magnetic field does not vanish in the presence of magnetic charge while in the absence of electric or magnetic charge the longitudinal component of the corresponding field vanishes. Thus the symmetry between electric and magnetic field is maintained by assuming the existence of magnetic charge.

Effects of generalized field equations

Imposing the generalized field equations (12) and (13) on the reduced expansion (11), we get the following conditions

$$\frac{e}{g} = \frac{j_0}{k_0} \quad (22)$$

$$g \quad (23)$$

These equations lead to the proportionality of electric charge and current densities with magnetic charge and current densities respectively, in terms of the ratio of electric and magnetic fundamental charges For simplicity, the restriction of constancy on the ratio of electric and magnetic charges on the same particle in the system may be imposed, *i.e.*,

$$\frac{e}{g} = k \text{ (constant)} \quad (24)$$

The effect of generalized field equations on the reduced expansions can be discussed in the following three cases

Case (*a*) In the absence of magnetic charge ($g = 0$, $e \neq 0$), the electric field is purely longitudinal while longitudinal magnetic field vanishes. In this case the reduced expansions of electric and magnetic fields are given by

$$\boldsymbol{E} = -i \sum_{\epsilon=\pm 1} c(\epsilon) \int \frac{d\boldsymbol{p}}{w(p)} \boldsymbol{p} f^0(\epsilon, \boldsymbol{p}) \exp[i\{\boldsymbol{p}.\boldsymbol{x} - \epsilon w(p)t\}]$$

$$+ i \sum_{\epsilon=\pm 1} c(\epsilon) \int d\boldsymbol{p} \exp[i\{\boldsymbol{p}.\boldsymbol{x} - \epsilon w(p)t\}] \left[m f^1(\epsilon, \boldsymbol{p}) + \frac{\boldsymbol{p}\{\boldsymbol{p}.f^1(\epsilon, \boldsymbol{p})\}}{w(p) + m} \right] \ldots \quad (25)$$

$$\boldsymbol{H} = im \sum_{\epsilon=\pm 1} c(\epsilon) \int \frac{d\boldsymbol{p}}{w(p)} \{\boldsymbol{p} \times f^1(\epsilon, \boldsymbol{p})\} \times \exp[i\{\boldsymbol{p}.\boldsymbol{x} - \epsilon w(p)t\}] \quad \ldots \quad (26)$$

which are similar to equations (1) and (2) except the additional spin-zero part. The values of $\boldsymbol{E}$ and $\boldsymbol{H}$ in this case may be specified by a single vector potential $\xi(x)$ as

$$\boldsymbol{H} = \nabla \times \xi(x)$$

$$\boldsymbol{E} = -\frac{\partial}{\partial t}\,\epsilon(x)$$

where vector $\xi(x)$ is given by

$$\xi(x) = -\sum_{\epsilon=\pm 1} c(\epsilon) \int \frac{d\boldsymbol{p}}{w(p)}\, \boldsymbol{p} f^0(\epsilon, \boldsymbol{p}) \exp[i\{\boldsymbol{p}\ \boldsymbol{x} - \epsilon w(p)t\}]$$

$$+ \sum_{\epsilon=\pm 1} c(\epsilon) \int \frac{d\boldsymbol{p}}{w(p)} \left[m f^1(\epsilon, \boldsymbol{p}) + \frac{\boldsymbol{p}\{\boldsymbol{p}\ f^1(\epsilon, \boldsymbol{p})\}}{w(p)+m} \right]$$

$$\times \exp[i\{p\ x - \epsilon w(p)t\}] \qquad \dots \quad (27)$$

From equations (2.5) and (26), we get,

$$\text{curl}\ E = -\sum_{\epsilon=\pm 1} c(\epsilon) cm \int d\boldsymbol{p} \exp i\{\boldsymbol{p}\ \boldsymbol{x} - \epsilon w(p)t\} \boldsymbol{p} \times f^1(\epsilon, \boldsymbol{p})\}$$

$$= -\frac{\partial \boldsymbol{H}}{\partial t} \qquad \dots \quad (28)$$

and

$$\text{div}\ \boldsymbol{H} = -\sum_{\epsilon=\pm 1} c(\epsilon) \int m \frac{d\boldsymbol{p}}{w(p)}\, p\ \{\boldsymbol{p} \times f^1(\epsilon, \boldsymbol{p})\} \exp(i\{\boldsymbol{p}\ \boldsymbol{x} - \epsilon w(p)t\})$$

$$= 0, \ [\text{since}\ \boldsymbol{p}.\{\boldsymbol{p} \times f^1(\epsilon, \boldsymbol{p})\} = 0] \qquad \dots \quad (29)$$

which shows that reduced expansions of electro-magnetic fields for non-zero mass system satisfy Maxwell's equations (28) and (29) in this case The other two Maxwell's equations may be used to find the values of charge source density j_0 and current source density $\boldsymbol{j}$ for the prescribed fields. These equations are given by

$$\text{div}\ \boldsymbol{E} = \sum_{\epsilon=\pm 1} c(\epsilon) \int d\boldsymbol{p} \exp[i\{\boldsymbol{p}\ \boldsymbol{x} - \epsilon w(p)t\}]$$

$$\times \left[\frac{p^2 f^0(\epsilon, \boldsymbol{p})}{w(p)} - \epsilon w(p)\{\boldsymbol{p}\ f^1(\epsilon, \boldsymbol{p})\} \right]$$

$$= 4\pi j_0 \qquad \dots \quad (30)$$

The charge density j_0 may be reduced o the irreducible representationsof proper, orthochronous, inhomogeneous Lorentz group according to the expansion (7) for scalar fields, where $f^0(\epsilon, \boldsymbol{p})$ is replaced by $j_0(\epsilon, \boldsymbol{p})$. Substituting this reduced expansion in eqn (30), we get,

$$[p^2 f^0(\epsilon, \boldsymbol{p}) - \epsilon w^2(p)\{p \cdot f^1(\epsilon, \boldsymbol{p})\}] = 4\pi j_0(\epsilon, \boldsymbol{p}) \qquad \dots \quad (31)$$

Similarly,

$$\Delta\times \boldsymbol{H}-\frac{\partial \boldsymbol{E}}{\partial t}=\sum_{\epsilon=\pm 1} c(\epsilon)\int \frac{\mathrm{d}\boldsymbol{p}}{w(p)}\exp[i\{\boldsymbol{p}.\boldsymbol{x}-\epsilon w(p)t\}]$$

$$\times\left[\boldsymbol{p}\epsilon f^0(\epsilon\,\boldsymbol{p})w(p)-m^3 f^1(\epsilon,p)-w(p)\boldsymbol{p}\{\boldsymbol{p}\,f^1(\epsilon,\boldsymbol{p})\left\{\frac{m}{w(p)}+\frac{w(p)}{m+w(p)}\right\}\right]$$

$$=4\pi \boldsymbol{j}. \quad \ldots \quad (32)$$

Current density vector $\boldsymbol{j}$ is reduced to the irreducible representations of proper, orthochronous, inhomogeneous Lorentz group by the similar expansion as given by eqn. (8) for a three-vector, where $f^0(\epsilon,\boldsymbol{p})$ and $f^1(\epsilon,\boldsymbol{p})$ are replaced by $j_0(\epsilon,\boldsymbol{p})$ and $j^1(\epsilon,\boldsymbol{p})$ This reduced expansion, when substituted in eqn. (32) gives.

$$\epsilon w(p)f^0(\epsilon,\boldsymbol{p})=4\pi j_0(\epsilon,\boldsymbol{p}) \quad \ldots \quad (33)$$

and

$$\left[m^3 f^1(\epsilon,\boldsymbol{p})+w(p)\boldsymbol{p}\{\boldsymbol{p}.f^1(\epsilon,\boldsymbol{p})\}\left\{\frac{m}{w(p)}+\frac{w(p)}{m+w(p)}\right\}\right]$$

$$=-4\pi\left[mj^1(c,\boldsymbol{p})+\frac{\boldsymbol{p}\{\boldsymbol{p}.j^1(\epsilon,\boldsymbol{p})\}}{w(p)+m}\right] \quad \ldots \quad (34)$$

From these equations one may find the value of mode $j(\epsilon,\boldsymbol{p})$ of current density for the prescribed electromagnetic fields with mode $f(\epsilon.\boldsymbol{p})$. In the absence of electric charge source ($j_0=0$) in this case, equation (33) gives,

$$f^0(\epsilon,\boldsymbol{p})=0 \quad \ldots \quad (35)$$

which when substituted in eqn. (31) gives,

$$p\,f^1(\epsilon,\boldsymbol{p})=0. \quad \ldots \quad (36)$$

If the current density is also zero in equation (34) besides the charge density then using eqn. (36) we get,

$$w^2(p)=p^2,\ i.e.,\ m=0 \quad \ldots \quad (37)$$

Therefore, the requirement that reduced expansion (25) and (26) satisfy Maxwell's eqn. (30) and (32) without sources gives $m=0$.

Case (*b*). In the absence of electric charge ($e=0$, $g\neq 0$) the reduced expansions of electric and magnetic fields are derived by a similar method. Here longitudinal part of electric field vanishes and a single vector opotential can describe those fields according to eqn. (27). These reduced expansions satisfy the field equations,

$$\text{div } \boldsymbol{E} = 0$$

and

$$\text{curl } \boldsymbol{H} = \frac{\partial \boldsymbol{E}}{\partial t}$$

From other two field equations one can calculate the values of magnetic charge and current source densities, k_0 and $\boldsymbol{k}$, for the prescribed fields in a similar manner as discussed for j_0 and $\boldsymbol{j}$ in the previous case, and it can be shown that in the absence of magnetic charge source the conditions (35) and (36) are satisfied If magnetic current density is also zero besides sharge density, then we get eqn. (37)

Case (c). In the presence of both electric and magnetic charges and current densities, effects of Maxwell's field equations are given by (22) and (23). The reduced expansions for J_0 and $\boldsymbol{J}$ can be written in the manner similar to equation (7) and (8) and the generalized densities can be calculated for the prescribed generalized electromagnetic fields

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